

Model Questions (Integral Calculus)

(Integration of Rational & Irrational Algebraic Functions)

1) (i) Integrate $\int \frac{dx}{(1+x)\sqrt{1-x^2}}$

(ii) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

2) (i) $\int \frac{x dx}{(a-x)(x-b)}$

(ii) $\int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$

3) (General Theorems on Definite Integral)

(i) Show that

$$\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$$

$$\int_0^{\pi/4} \log(1+\tan \theta) d\theta = \frac{\pi}{8} \log 2$$

(ii) Evaluate $\int_0^{\pi/2} \log \sin x dx$

4) Reduction Formula

(i) Evaluate $\int_0^{\pi/4} \tan^5 x dx$

(ii) Evaluate $\int_0^{\infty} x^n e^{-x} dx$

5 (i) Evaluate $\int_0^{\pi/2} \cos^n x \cdot \cos nx dx$

(ii) If n is a positive integer > 1 , prove that

$$\int_0^{\pi/2} \cos^{n-2} x \cdot \sin nx dx = \frac{1}{(n-1)}$$

6) Differentiation under the sign of Integration.

(i) Evaluate $\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)} dx$ where $a > 1$.

(ii) Evaluate $\int_0^{\infty} \frac{\log(1+a^2x^2)}{1+b^2x^2} dx$

7) Curve Tracing

(i) Trace the curve $x^3 + y^3 = 3axy$ (Folium of Descartes) (Cartesian Curves)

(ii) $x^{2/3} + y^{2/3} = a^{2/3}$ (Astroid)

(iii) $r = a(1 + \cos \theta)$ Cardioid
(iv) $r^2 = a^2 \cos 2\theta$ (Lemniscate of Bernoulli) } Polar Curves

8) Area of Curves

(i) Find the whole area of the curve $a^2y^2 = x^2(a^2 - x^2)$.

(ii) Trace and find the loop of the curve $x^3 + y^3 = 3axy$.

(9) (i) Find the area of the Cardioid $r = a(1 + \cos \theta)$

(ii) Find the area of the loop of the curve $r^2 = a^2 \cos 2\theta$.

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⑩ Length of the plane Curves (Cartesian Curves)

(i) Find the perimeter of the Curve

$$x^{2/3} + y^{2/3} = a^{2/3}.$$

(ii) Find the whole length of the loop of the curve

$$3ay^2 = x(x-a)^2.$$

⑪ Length of Plane Curves (Polar Curves)

(i) Prove that the total length of the cardioid $r = a(1 - \cos \theta)$ is $8a$.

Also show that the arc above the initial line is bisected by the line $\theta = \frac{2\pi}{3}$.

(ii) Find the length of the arc of the curve $\frac{2a}{r} = 1 + \cos \theta$ from $\theta = 0$ to

$$\theta = \frac{\pi}{2}.$$

⑫ The volume and Surface Area of the Solids of Revolution.

(i) Find the volume and Surface area of a sphere of radius a .

(ii) Find the volume and Surface area of the solid generated by the revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x -axis.